Review Essays

The Space of Music: Review Essay of Dmitri Tymoczko's *A Geometry of Music*

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In recent decades we have seen a gradual shift of emphasis in academic musicology, away from the study of the great tradition of Western art music to the empirical investigation of the musical ear. The rise of cognitive neuroscience has given impetus to this shift. For it reminds us that music is not sound, but sound organized "in the brain of the beholder." Musical organization is something that we "latch on to," as we latch on to language. And once the first steps in musical comprehension have been taken, we advance rapidly to the point where each of us can immediately absorb and take pleasure in an indefinite number of new musical experiences. This recalls a fundamental feature of language, and unsurprisingly, results from linguistics have been transferred and adapted to the analysis of musical structure in the hope of showing just how it is that musical order is generated and perceived, and just what it is that explains the grip that music exerts over its devotees.

We should recognize here that music is not just an art of sound. We might combine sounds in sequence as we combine colors on an abstract canvas, or flowers in a flowerbed. But the result will not be music. It becomes music only if it also makes *musical sense*. Leaving modernist experiments aside, there is an audible distinction between music and mere sequences of sounds, and it is not just a distinction between *types* of sound (e.g. pitched and unpitched, regular and random). Sounds become music as a result of organization, and this organization is something that we perceive and whose absence we immediately notice, regardless of whether we take pleasure in the result. This organization is not just an aesthetic matter, not simply a *style*. It is more like a grammar, in being the precondition of our response to the result *as music*. We must therefore acknowledge that tonal music has something like a syntax—a rule-guided process linking each episode to its neighbors, which we grasp in the act of hearing, and the absence of which leads to a sense of discomfort or incongruity.

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Of course there are things called music which do not share this syntax, for example, modernist experiments, African drum music, music employing scales that defy harmonic ordering, and so on. But from medieval plainsong to modern jazz we observe a remarkable constancy, in rhythmical, melodic, and harmonic organization, so much so that one extended part of this tradition has been singled out as "the common practice" whose principles are taught as a matter of course in classes of music appreciation. This phenomenon demands an explanation.

Leonard B. Meyer argues that we understand music by a kind of probabilistic reasoning, which endows musical events with varying degrees of redundancy.[1] The common practice has emerged from a steady accumulation of conventions and expectations, which enable listeners to predict what follows from what, and which give rise to the distinctive "wrong note" experience when things go noticeably astray. This suggestion was taken forward by Eugene Narmour, to produce what he calls the "implication-realization model" of musical structure.[2] And more recently, David Temperley has applied Bayesian probability theory to standard rhythms and melodies, in order to "model" the way in which listeners assign meter and tonality to sequences.[3] Temperley's work raises three questions: What is a "model"? When is a model "adequate" to the data? And what might the discovery of an adequate model show, concerning our understanding and appreciation of music? A model that can be rewritten as an algorithm could program a computer to recognize (or should we say "recognize"?) metrical order and key. Such a model can be tested against human performance, and if it successfully predicts our preferences and decisions, it offers the beginning of a theory of musical cognition. It suggests an account of what goes on in the brain, when a listener identifies the metrical and tonal structure of the piece he is listening to. And that seems to be the aim of Temperley's reflections.

However, others use the term "model" more loosely, to mean any way of representing the musical surface that displays the perceived connections among its parts, and which suggests a way in which we grasp those connections, whether consciously or not. In this sense the circle of fifths, chord-sequence analysis, and the old charts of key relations are all partial "models'" of our musical experience. They enable us to predict, up to a point, how people will respond to changes of key and to accidentals in a melody, and they also suggest musical "constants" on which a composer can lean when constructing the harmonic framework of a piece. But they do not

[1] Leonard B. Meyer, *Emotion and Meaning in Music* (Chicago, IL: University of Chicago Press, 1956).

[2] Eugene Narmour, *The Analysis and Cognition of Basic Melodic Structures* (Chicago, IL: University of Chicago Press, 1990).

[3] David Temperley, *Music and Probability* (Cambridge, MA: MIT Press, 2007).

aim to reduce musical understanding to a computational algorithm, nor do they offer anything like a complete theory of musical cognition that will explain how we assemble a coherent musical surface from our experience of its parts. Rather, they describe the surface, by identifying the salient features and the perceived relations between them.

Things would look a little different, however, if we could take the idea of a musical "syntax" literally. Linguistics attempts to model language use and comprehension in ways that lend themselves to computational analysis. If we could extend to the realm of musicology the advances made in psycholinguistics, therefore, we might be nearer to explaining what goes on, when people assemble the notes that they hear into coherent structures. Inconclusive research by neuroscientists suggests that "although musical and linguistic syntax have distinct and domain-specific syntactic representations, there is overlap in the neural resources that serve to activate these representations during syntactic processing."[4] This—"the shared syntactic integration resource hypothesis"—would be of considerable interest not only to evolutionary psychology but also to musicology, if it could be shown that the syntactic processes involved in the two cases work in a similar way. The neurological research does not show this. But there is a kind of speculative cognitive science that suggests that it might nevertheless be true, and that a "grammar" of tonal music could be developed which both resembles the grammar of language and can also be rewritten as a computational algorithm.

One goal of Noam Chomsky's generative grammar has been to explain how speakers can understand indefinitely many new utterances, despite receiving only finite information from their surroundings.[5] Formal languages like the predicate calculus provide a useful clue, showing how infinitely many well-formed formulas can be derived by recursion. If natural languages are organized in the same way, then from a finite number of basic structures, using a finite number of transformation rules, an infinite number of well-formed sentences could be extracted. Understanding a new sentence would not be a mystery, if speakers were able to recuperate from the string of uttered words the rule-governed process that produced it. Likewise, the widespread capacity to latch on to new music without any guidance other than that already absorbed through the ear, could be explained if musical surfaces were the rule-governed products of a finite number of basic structures, which might be partly innate and partly acquired during the early years of acculturation.

Certain aspects of music have been modeled in ways that suggest such a generative grammar. If metrical organization proceeds by division, as in Western musical systems, then surface rhythms can be derived from basic

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[4] Aniruddh D. Patel, *Music, Language and the Brain* (Oxford: Oxford University Press, 2008), p. 297.

[5] Noam Chomsky, *Syntactic Structures*, 2nd ed. (Berlin: Walter de Gruyter, 2002).

structures by recursion and also understood by recuperating that process. This is made into the basis of a generative grammar of metrical rhythm by Christopher Longuet-Higgins and C. S. Lee.[6] Others have made similar first shots at grammars for pitch organization.[7]

Such small-scale proposals were quickly displaced by the far more ambitious theory presented by Fred Lerdahl and Ray Jackendoff in their ground-breaking book, *A Generative Theory of Tonal Music*.[8] Their argument is bold, ambitious, and detailed. But they recognize at many points that the analogy with language is tenuous, and that Chomskian linguistics cannot be carried over wholesale into the study of tonal music. For one thing, the hierarchical organization that Lerdahl and Jackendoff propose is an organization of individual musical objects, such as notes and chords, and not, as in Chomsky, of grammatical categories (verb, noun-phrase, adverb, etc.). There are no grammatical categories in music. Moreover, while we can distinguish "structural" from "subordinate" events in music, there is much room for argument as to which is which, and there is no one hierarchy that determines the position of any particular event. An event that is structural from the "time-span" point of view might be metrically subordinate and also a prolongation of some other event in the hierarchy of tension and release. Still, the various hierarchies identified by Lerdahl and Jackendoff capture some of our firmer intuitions about musical importance. The task is to show that there are transformation rules that derive the structure that we hear from a more deeply embedded structure, and do so in such a way as to explain our overall sense of the connectedness of the musical surface.

In order to grasp the point of the generative theory of tonal music it is important to distinguish two kinds of hierarchy: generative and cumulative. A generative hierarchy is one in which structures at the level of perception are generated from structures at the "higher" level by a series of rule-governed transformations. Perceivers understand the lower-level structures by unconsciously recuperating the process that created them, "tracing back" what they see or hear to its generative source. By contrast, a cumulative hierarchy is one in which perceived structures are repeated at different temporal or structural levels, but in which it is not necessary to grasp the higher level in order to understand the lower. For example, in classical architecture, a columnated entrance might be contained within a façade that exactly replicates its proportions and details on a larger scale. Many architectural effects are achieved in that way, by the "nesting" of one aedicule within

[6] Christopher Longuet-Higgins and C. S. Lee, "The Rhythmic Interpretation of Monophonic Music," *Music Perception* 1, no. 4 (Summer 1984), pp. 424-41.

[7] For example, Diana Deutsch and John Feroe, "The Internal Representation of Pitch Sequences in Tonal Music," *Psychological Review* 88, no. 6 (1981), pp. 503-22.

[8] Fred Lerdahl and Ray Jackendoff, *A Generative Theory of Tonal Music* (Cambridge, MA: MIT Press, 1983).

another, so that the order radiates outward from the smallest unit across the façade of the building. This is not an instance of "generative" grammar in the sense that this term has been used in linguistics, but rather of the amplification and repetition of a separately intelligible design. It is true that the order of such a façade is generated by a rule, namely, "repeat at each higher scale," but we understand each scalar level in the same way as every other. You recognize the pattern of the entrance, and you recognize the same pattern repeated on a larger scale in the façade. Neither act of recognition is more basic than the other, and neither depends on the other. In my *The Aesthetics of Music*, I argue that many of the hierarchies discerned in music, notably the rhythmic hierarchies described by Grosvenor Cooper and Leonard B. Meyer,[9] are cumulative rather than generative, and therefore not understood by tracing them to some hypothetical "source."[10] In the case of rhythm there are generative hierarchies, too, as was shown by Longuet-Higgins, writing at about the same time as Cooper and Meyer. But it seems to me that, in the haste to squeeze music into the framework suggested by linguistics, writers have not always been careful to distinguish the two kinds of hierarchy. Music, in my view, is more like architecture than it is like language, and this means that repetition, amplification, diminution, and augmentation have more importance in creating the musical surface than do rule-guided transformations of some structural "source."

The place of semantics in the generation of surface syntax is disputed among linguists, and Chomsky has not adhered to any consistent view in the matter. As a philosopher, however, influenced by a tradition of thinking that reaches from Aristotle to Gottlob Frege and Alfred Tarski and beyond, I would be surprised to learn that deep structure and semantics have no intrinsic connection. Language, it seems to me, is organized by generative rules not by chance, but because that is the only way in which it can fulfill its primary function of conveying information. Deep structures must surely be semantically pregnant if the generative syntax is to shape the language as an information-carrying medium, one in which new information can be encoded and received. Without deep structure, language would surely not be able to "track the truth," nor would it give scope for the intricate question-and-answer of normal dialogue. A syntax that generates surface structures from deep structures is the vehicle of meaning, and that is why it emerged.

Take away the semantic dimension, however, and it is hard to see what cognitive gain there can be from a syntax of that kind. In particular, why should it be an aid to comprehension that the syntactical rules generate surface structures out of concealed deep structures? This question weighs heavily on

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[9] Grosvenor Cooper and Leonard B. Meyer, *The Rhythmic Structure of Music* (Chicago, IL: University of Chicago Press, 1960).

[10] Roger Scruton, *The Aesthetics of Music* (Oxford: Oxford University Press, 1997), p. 33.

the generative theory of music, precisely because music is not "about" anything, either in the way that language is about things or in the way that figurative painting is about things. Indeed, musical organization is at its most clearly perceivable and enjoyable in those works, like the fugues of Bach and the sonata movements of Mozart, which are understood as "abstract" or "absolute," carrying no reference to anything beyond themselves.

You might say that a hierarchical syntax would facilitate the ability to absorb new pieces. But this ability is as well facilitated by rules that operate on the surface, in the manner of the old rules of harmony and counterpoint or by the techniques of local variation and embellishment familiar to jazz improvisers. What exactly would be added by a hierarchical syntax, that is not already there in the perceived order of repetition, variation, diminution, augmentation, transposition, and so on? Perhaps it is only in the case of metrical organization that a generative hierarchy serves a clear musical purpose, since (in Western music at least) music is measured out by division, and divisions are understood by reference to the larger units from which they derive.

There is a theory, that of Heinrich Schenker, which offers to show that harmonic and melodic organization are also hierarchical, and Lerdahl and Jackendoff acknowledge their indebtedness to this theory.[11] According to Schenker, tonal music in our classical tradition is (or ought to be) organized in such a way that the musical surface is derived by "composing out" a basic harmonic and scalar progression. This basic progression provides the background, with postulated "middle ground" structures forming the bridges that link background to foreground in a rule-governed way. Musical understanding consists in recuperating at the unconscious level the process whereby the background *Ursatz* (or fundamental structure) exfoliates in the musical surface.

Objections to Schenker's idea are now familiar. Not only does it reduce all classical works, or at least all classical masterpieces, to a single basic gesture. It also implies formidable powers of concentration on the listener's part, to hold in suspension the sparse points at which the *Ursatz* can be glimpsed beneath the surface of a complex melodic and harmonic process. Moreover, it leaves entirely mysterious what the *benefit* might be, either in composing or in listening to a piece, the understanding of which involves recuperating these elementary musical sequences that have no significance when heard on their own.

More importantly, the whole attempt to transfer the thinking behind transformational grammar to the world of music is a kind of *ignoratio elenchi*. If music were like language in the relevant respects, then grasp of musical

[11] Heinrich Schenker, *Free Composition*, ed. and trans. Ernst Oster (New York: Longman Press, 1979). For an exposition of Schenker's analysis, see Felix Salzer, *Structural Hearing: Tonal Coherence in Music*, two vols. (Mineola, NY: Dover Publications, 1982 [1952]).

grammar ought to involve an ability to produce new utterances, and not just an ability to understand them when produced by someone else. But there is a striking asymmetry here. All musical people quickly "latch on" to the art of musical appreciation. Very few are able to compose meaningful or even syntactically acceptable music. It seems that musical understanding is a one-way process, and musical creation a rare gift that involves quite different capacities from those involved in appreciating the result.

Here we discover another difficulty for theories like that of Lerdahl and Jackendoff, which is that they attempt to cast what seems to be a form of aesthetic preference in terms borrowed from a theory of truth-directed cognition. If understanding music involves recuperating information (either about the music or about the world), then a generative syntax would have a function. It would guide us to the semantically organized essence of a piece of music, so that we could understand what it says. But if music says nothing, why should it be organized in such a way? What matters is not semantic value but the agreeableness of the musical surface. Music addresses our preferences, and it appeals to us by presenting a heard order that leads us to say "yes" to this sequence, and "no" to that. Not surprisingly, therefore, when Lerdahl and Jackendoff try to provide what they regard as transformation rules for their musical grammar, they come up with "preference rules," rather than rules of well-formedness. These "rules" tell us, for example, to "prefer" to hear a musical sequence in such a way that metrical prominence and time-span prominence coincide. There are some one hundred of these rules, which, on examination, can be seen not to be rules at all, since they do not owe their validity to convention. They are generalizations from the accumulated preferences of musical listeners, which are not guides to hearing but by-products of our musical choices. Many of them encapsulate aesthetic regularities, whose authority is stylistic rather than grammatical, like the norms of poetic usage.

The formal languages studied in logic suggest, to a philosopher at any rate, what might be involved in a generative grammar of a natural language: namely, rules that generate indefinitely many well-formed strings from a finite number of elements, and rules that assign semantic values to sentences on the basis of an assignment of values to their parts. Nobody, I believe, has yet provided such a grammar for a natural language. But everything we know about language suggests that rules distinguishing well-formed from ill-formed sequences are fundamental, and that these rules are not generalizations from preferences but conventions that define what speakers are doing. They are what John Searle calls "constitutive" rules.[12] Such rules have a place in tonal music, for example, the rule that designated pitches come from a set of twelve octave-equivalent semitones. But they do not seem to be linked to a generative grammar of the kind postulated by Lerdahl and Jackendoff. They simply lay down the constraints within which a

[12] John Searle, *The Construction of Social Reality* (New York: Free Press, 1997).

sequence of sounds will be heard as music, and outside of which it will be heard as non-musical sound. Moreover, these constitutive rules are few and far between, and far less important, when it comes to saying how music works than are the résumés of practice that have been studied in courses of harmony and counterpoint.

This brings me to the crux of the issue. There is no doubt that music is something that we can understand and fail to understand. But the purpose of listening is not to decipher messages, or to trace the sounds we hear to some generative structure, still less to recuperate the information that is encoded in them. The purpose is for the listener to follow the musical journey, as rhythm, melody, and harmony unfold according to their own inner logic, so as to make audible patterns linking part to part. We understand music as an object of aesthetic interest, and this is quite unlike the understanding that we direct toward the day-to-day utterances of a language, even if it sometimes looks as though we "group" the elements in musical space in a way that resembles our grouping of words in a sentence.

This does not mean that there is no aspect to musical grammar that would deserve the sobriquet "deep." On the contrary, we recognize long-term tonal relations, relations of dependence between episodes, ways in which one part spells out and realizes what has been foretold in another. These aspects of music are important: they are the foundation of our deepest musical experiences and an endless source of curiosity and delight. But they concern structures and relations that are created in the surface, not hidden in the depths. The musical order is not generated *from* these long-term relations, as Schenker would have us believe, but points *toward* them, in the way that architectural patterns point toward the form in which they culminate. We come to understand the larger structure as a result of understanding the small-scale movement from which it derives.

One of the strengths of Lerdahl and Jackendoff's *A Generative Theory of Tonal Music* is that it emphasizes these long-term relations, and the way in which the listener—especially the listener to the masterworks of our listening culture—hears the music as *going somewhere*, fulfilling at a later stage expectations subliminally aroused at an earlier one. The mistake, it seems to me, comes from thinking that these perceived relations define a hidden or more basic structure, from which the rest of the musical surface is derived. The perceived relations should rather be seen as we see the relation between spires on a Gothic castle. The pattern made by the spires emerges *from* the supporting structures, but does not generate them.

In a formidable recent work, musicologist and mathematician Dmitri Tymoczko argues that the common practice of Western classical music is really just one section of an "extended common practice" that stretches from early medieval times down to modern jazz, pop, and such concert-hall music as pleases the normal musical ear.[13] And it is the existence of this extended

[13] Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011). Page

common practice that gives credibility to the hypothesis that there is a unified generative grammar of tonal music. If we think that there is a process of "grasping" musical order that is somehow prior to and necessary for aesthetic understanding, and if this process engages with deeply embedded cognitive capacities, then this would explain the longevity and seeming naturalness of the extended common practice. It would also explain such otherwise remarkable facts as these: that Western music, whether classical or pop, is of global appeal, and has a lamentable tendency to drive out the native music of every place where it is introduced; that works of music are easily memorized both by listeners and performers; that those with a knowledge of the common-practice tradition can assign a previously unheard work with the greatest precision to its date (that is, to its point of syntactical development); that the chords and scales of concert-hall music reappear in popular music, often embedded in similar harmonic networks.

Tymoczko is, for good reasons, unpersuaded by the analogy between musical and linguistic comprehension. Nevertheless, his theory resembles the "generative theory of tonal music" in one important respect, which is that it offers to explain the observable in terms of the hidden. Tymoczko accounts for our intuitive ability to latch on to musical sequences by reference to an arcane geometry that arranges musical objects in another space from the one in which we hear them. Somehow, this "geometry of music" is supposed to be what we are mentally exploring when we hear the rightness of chord progressions and the persuasive nature of voice-leadings. Music is not, as Gottfried Wilhelm Leibniz famously said, a matter of unconscious arithmetic, but more a matter of unconscious geometry.[14] (As for Arthur Schopenhauer's view of music, as "unconscious metaphysics,"[15] this no longer gets a look in.)

The attempt to arrange musical relations in a geometrical model is by no means new. Circles, maps, and spirals modeling root progressions and key shifts are associated with such names as Johan Mattheson, David Kellner, Gottfried Weber, and Leonhard Euler, and played a large part in the great explosion of music theory in the eighteenth century. More recently, Longuet-Higgins has developed a geometrical model of tonal relations, and Lerdahl, in a formidably difficult work, has recast the findings of *A Generative Theory of Tonal Music* in terms of paths taken through "tonal pitch space," although, as he here and there acknowledges, his model is numerical rather than spatial, and talk of "regions" of "pitch space" involves a kind of metaphor.[16]

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references are in parentheses in the text.

[14] Gottfried Wilhelm Leibniz, "Letter to Christian Goldbach, April 17, 1712," in *Epistolae Ad Diversos*, ed. Christian Kortholt, vol. 1 (Leipzig: Breitkopf, 1734), p. 240.

[15] Arthur Schopenhauer, *The World as Will and Representation*, ed. and trans. E. F. J. Payne, vol. 1 (New York: Dover, 1969), sec. 52.

[16] Fred Lerdahl, *Tonal Pitch Space* (Oxford: Oxford University Press, 2001).

However, Tymoczko takes the idea of a musical geometry forward in a novel way, by proposing a complete account of voice-leading and harmonic progression, and mounting a kind of *a priori* argument for the naturalness of the "extended common practice," by which he means, essentially, Western music from plainsong to pop (p. 27).

Tymoczko begins from a pre-theoretical conception of tonal music, in terms of five features that are so familiar to us that we find it hard to define them precisely. Tonal music shows a preference for "conjunct melodic motion" (that is, small intervals and fluent movement across them); it exhibits a widespread use of "acoustic consonance," with octave, fifth, and fourth assuming prominent melodic and harmonic roles; there is a tendency to "harmonic consistency" (consonant sequences or dissonant sequences, but not a scrambled mixture of both); pitches are organized as scales within the octave; and certain notes are singled out as more important or central than others (for instance, the tonic, the dominant, the leading note) (p. 4). Tymoczko's description of these features is loose, although they serve as his definition of tonality, and are probably no more deficient as a definition than other attempts to pin down a phenomenon that is as elusive to the intellect as it is familiar to the ear. Tymoczko's purpose in *The Geometry of Music* is to explain and vindicate four claims about tonal music, so defined.

The first claim is that harmony and counterpoint constrain one another, so that harmony cannot be understood independently of the voice-leading that generates each chord from its predecessor. The second claim is that scale, macroharmony (which is the total collection of notes used over small stretches of musical time), and centricity are independent. In other words, a piece might be centered around a given pitch class (but use scales that do not identify that pitch class as the tonic) and notes that have no designated function in the given key. The third claim is that modulation tends to involve what Tymoczko calls "efficient" voice-leading, in which voices tend to move by scale steps or semitones (pp. 11-19).

Those three claims are, in my view, true, and the strongest aspect of Tymoczko's book is the case that he gives for voice-leading in the common practice. He makes abundantly clear, both theoretically and through detailed examples, that real musicians in the tonal tradition think of chords not as pitch-class sets but as structures emerging from the movement of voices. This is as true of jazz as it is true of Bach's fugues or Mozart's symphonies. It explains why Berg's Violin Concerto is so popular—namely, that the harmonies (notwithstanding their atonal character) are almost entirely derived by voice-leading, whether or not they also conform to the permutational calculus of pitch classes which supposedly organizes the piece.

In 1973 Allen Forte published his highly influential book, *The Structure of Atonal Music*, in which he develops a set-theoretic analysis of serial music.[17] Forte's approach involves rewriting "simultaneities" as pitch-

[17] Allen Forte, *The Structure of Atonal Music* (New Haven, CT: Yale University Press,

class sets and reducing them to their "normal" ordering, with intervals arranged to be as short as octave equivalence allows. This clever book, the influence of which can be discerned in many subsequent academic studies, did an enormous disservice to musicology. For it describes harmony while entirely ignoring voice-leading, which is the vehicle of harmonic progression and therefore an integral component of harmonic meaning even in atonal chords.[18] Maybe it is true in some works of serial music that voice-leading has no role, and maybe that is why we hear the result not as "harmony" but as "simultaneity." But that is exactly what leads us to resist that kind of serial music and why it will never have a place in ordinary musical affections. By describing harmonies in Forte's way you deprive yourself of an instrument of musical criticism. You also ignore a whole dimension of musical understanding, a dimension that Tymoczko works hard to make central to the nature and meaning of tonal music. As he shows, the basic sonorities of Western tonal music arise from efficient voice-leading, harmonic consistency, and acoustic consonance, and these three features are woven together in the extended common practice. That, in a nutshell, is why "tonality rules OK."

Forte's nonsensical account of atonal music issues from an earlier attempt to explain musical understanding mathematically, using modulo twelve arithmetic to model pitch-class sequences and simultaneities. For Tymoczko it is not arithmetic but geometry that contains the secret, and his fourth claim is that "music can be understood geometrically." Or rather, as he instantly explains, "geometry provides a powerful tool for modelling musical structure" (p. 19). Those two statements are not equivalent, however, and Tymoczko never makes entirely clear which of them he wishes to insist upon. Moreover, in the sense that he intends them, neither claim is true.

If you take the notion of a model in the loose manner that I above remarked on, then many things that we do not understand geometrically can be provided with geometrical models. You can model a game of soccer by a path evolving in forty-six dimensions (two dimensions for each of the twenty-two players and two for the ball), but the result will not help you to understand or play a game of soccer, since it is derived from moves that we recognize in another way, and adds nothing to our ability to decide or predict them. The geometry is a shadow cast in forty-six-dimensional space by the light of intuitive practice. Even if we can model the chords of tonal harmony in an "ordered pitch space," in such a way as to represent the efficient voice-leadings between them, this too may be no more than a shadow cast by a practice that we understand in another way. Tymoczko's "tool for modelling musical structure" would be "powerful" only if it either were to add to our understanding of music or to suggest an explanation of how musical elements

1973).

[18] See the discussion of Berg's Violin Concerto in Scruton, *The Aesthetics of Music*, pp. 301-2.

are processed in the brain. But, after wrestling for painful hours with his "ordered pitch spaces," in which chords are assembled in relation to their standard transformations on an infinite Möbius band, I came to the conclusion that this "geometry of music" is exceedingly clever but more or less irrelevant. I was confirmed in this conclusion by Tymoczko's own critical studies in the second part of the book where, with very few exceptions, he explains his interesting ideas concerning voice-leading, chromaticism, and scalar organization more or less entirely in traditional analytical language, using old-fashioned chord grammar and setting out the passages to be explained not in his *n*-dimensional pitch space, but in ordinary musical notation. When expounding his geometry, he writes that

> learning the art of musical analysis is largely a matter of learning to overlook the redundancies and inefficiencies of ordinary musical notation. Our geometrical space simplifies this process, stripping away musical details and allowing us to gaze directly upon the harmonic and contrapuntal relationships that underlie much of Western contrapuntal practice (p. 79).

He makes this point in the context of an analysis of a few bars from a Brahms Intermezzo, giving both a complex geometrical graph and the relevant bits of the score. The graphs are all but unintelligible, but through the score you "gaze directly" on the notes, and the score offers all the reader needs in order to grasp Tymoczko's argument.

I make this point with some hesitation, being impressed by Tymoczko's singular combination of mathematical knowledge and musical insight. But it is perhaps worth pointing out that his geometry of chord progressions and harmonic relations was anticipated by Longuet-Higgins.[19] Longuet-Higgins introduced a three-dimensional tonal space, with octaves assigned to one dimension, fifths to another, and thirds to another. All of the intervals in tonal music can be defined on this space, in which they appear as vectors. Moreover, and this particularly interested Longuet-Higgins, this tonal space distinguishes between intervals that are indistinguishable from the point of view of modulo twelve arithmetic. Thus, it distinguishes between a major third and a diminished fourth, for example, even though they are both (in well-tempered scales) made up of four equal semitones. The tonal space displays the real, hidden, grammar of tonal music, since it preserves the scalar

[19] Christopher Longuet-Higgins, "Two Letters to a Musical Friend," *The Music Review* 23 (1962), pp. 244-48 and 271-80, reprinted in Christopher Longuet-Higgins, *Mental Processes: Studies in Cognitive Science* (Cambridge, MA: MIT Press, 1987), pp. 64-81. Longuet-Higgins, a theoretical chemist by training, was a brilliant musician, who invented the term "cognitive science" and who did as much as anyone else to set up the discipline to which that term now refers; he was at least as multi-competent as Tymoczko.

meaning of the intervals in their harmonic representation. A succession of triads defines a path in this space, and this path may either hop around a center, in which case the music remains in one key, or move from one center to another, in which case there has been a modulation to another key. Longuet-Higgins gives a precise definition that distinguishes these two cases, and uses it to assign notation to difficult examples of highly chromatic pitches, such as the *cor anglais* solo in the introduction to the third act of *Tristan*. The geometry used by Longuet-Higgins does not emphasize voice-leading as Tymoczko does, but in other respects it applies the same intuitive idea, namely, that musical relations can be mapped onto geometrical relations by preserving "betweenness." It also looks very much like the first step in an explanatory theory, suggesting a way in which the brain "maps" the musical input, as the visual system maps orientation, distance, etc., so as to represent edges, discontinuities, and occlusions. Here is one of Longuet-Higgins's typically laconic summaries:

> The three-dimensionality of tonal space follows directly from the fact that just three basic intervals are necessary and sufficient for the construction of all others. Given any note such as middle C we may place it at the origin in tonal space and relate all other notes to it by assigning them coordinates (*x*,*y*,*z*) which represent the numbers of perfect fifths, major thirds and octaves by which one must move in order to get from middle C to the note in question. In principle, then, the notes of tonal music lie at the points of a discrete three-dimensional space which extends infinitely in all directions away from any starting point. Viewed in this way, the notes of a melody perform a "dance" in an abstract conceptual space; the appreciation of tonality depends upon the ability to discern the direction and distance of each step in the dance.[20]

Tymoczko does not mention Longuet-Higgins, who nevertheless deserves credit for his lapidary articles, which take only a few pages (compared to some 150 pages of Tymoczko) to show how to represent musical relations geometrically. But there is an important point to be made in response to both writers, which is that we already have an idea of musical space, which is quite unlike the geometrical orderings set forth in their studies. We hear music as a kind of movement in one-dimensional space. This space is ordered in terms of a betweenness relation defined on the axis of pitch (the axis of "high" and "low"). It has interesting topological features, for example, most three-note chords cannot be transposed onto their mirror images. It is folded over at the octave, so that movement in one direction returns to the same place after twelve semi-tone steps. In its musical use it is endowed with

[20] Longuet-Higgins, "The Grammar of Music," in Longuet-Higgins, *Mental Processes*, p. 140.

gravitational fields of force, according to scalar measure and key relations. The leading note is drawn toward the tonic; dominant seventh chords tend toward tonic chords, and so on. But this space is a purely phenomenal space. No musical object can be identified except in terms of its place (middle C, for instance), so that position in musical space is an essential property of whatever possesses it. Hence, although we hear movement, nothing moves. The space that we hear is a kind of metaphorical space, but one that is vividly etched on our auditory experience. Moreover, it is a space that contains interesting symmetries and which can be treated mathematically in ways that cast light on our musical experience.[21]

Tymoczko's "process-based" approach to chromaticism, which emphasizes voice-leading as opposed to static chords, is persuasive, largely because he takes us on journeys through this phenomenal space, rarely troubling to look behind him, at the spooky shadows cast on those Möbius bands. Standard clef notation represents the phenomenal space of music with all of the clarity and detail that a critic needs, and when a critic tells us that the G-sharp of the "Tristan chord" moves chromatically to B while the D-sharp moves to D, he describes exactly what we hear as well as what we see on the page—even though the description is literally speaking nonsense, since G-sharp cannot move to B nor D-sharp to D. Moreover, the one-dimensional space of standard notation reminds us of a fact that Tymoczko rarely adverts to, namely, that voice-leading is not merely a matter of relations between adjacent notes and adjacent chords. It runs through a whole piece of music, creating expectations in each note that reach well beyond its immediate successor. The Prelude to *Tristan* is a wonderful example of this. It does not merely proceed from one unsaturated harmony to the next; each voice pursues its own lonely anxiety-ridden journey through tonal space, moving by semi-tone or whole-tone steps in obedience to a kind of obsession that seems to owe nothing to the harmonic network of which it is a part.

Tymoczko rightly comments on the way in which the circle of fifths gave way, in romantic music, to the circle of thirds, and he offers sensitive and persuasive accounts of the way in which third-relations in Schubert and Chopin arise from chromatic voice-leading. But these accounts rely on our intuitive understanding of the one-dimensional phenomenal space of music, and the gravitational tensions implanted in that space by scales and chords. Tymoczko tells us that "the geometry of chord space ensures that (Schubert's and Chopin's) sort of intuitive exploration will necessarily result in music that can be described using the major- and minor-third systems," and he adds that

[21] For an example of this, see Wilfrid Hodges, "The Geometry of Music," in *Music and Mathematics: From Pythagoras to Fractals*, ed. John Fauvel, Raymond Flood, and Robin Wilson (Oxford: Oxford University Press, 2003), pp. 90-111. As Hodges shows, there is another and more useful geometry of music, which describes the phenomenal space in which music exists, rather than the imaginary *n*-dimensional "model" which breathlessly tries to keep track of it.

his "geometrical spaces . . . are literally the terrain through which chromatic music moves" (p. 220). Both claims are surely unjustified. For one thing, chromatic music does not literally move through any space at all, and it metaphorically moves through the one-dimensional space identified by the ordinary musical ear. Tymoczko's description of the way in which triads are smoothly connected by chromatic voice-leading to their major-third transpositions and seventh chords to their minor-third and tritone transpositions is a description of movements and relations in phenomenal space. And when he adds that his "geometrical spaces . . . offer a convenient way to visualize these facts" (p. 220), he in effect concedes that he has not given an explanation, but only a model in the loose sense of that term mentioned above. The question then arises how this model might be used: Is it the first step in a cognitive science of music, such as Longuet-Higgins wishes to provide? If so, what would be the neural correlate of the infinite Möbius strip? Here we come up against a brick wall. We can translate tonal music into a kind of geometry. And we can understand how computations can combine variables in more than one dimension. But how do we get from the geometrical models to the computations in the brain?

If we adhere to the strict sense of "model" that I referred to at the start of this review, according to which a model is the first step toward a computational algorithm, then it is clear that no model can make use of the phenomenal space that is described by ordinary musical notation. A space in which position, movement, orientation, and weight are all metaphors is not a space that can feature in a computer program, or indeed in any kind of theory that seeks to explain our experience rather than to describe its subjective character. But here is a point at which the defenders of old-fashioned musicology might wish to step in with a long suppressed protest. Musicology, they might say, is a form of humanistic study and not a science. It cannot be replaced by mathematical analysis, nor is it a prelude to a theory of musical cognition, whatever that may be. It is devoted to describing, evaluating, and amplifying the *given* character of musical experience, rather than to showing how musical preferences might be tracked by a computer. Hence the one-dimensional pitch space in which *we*, self-conscious and aesthetically motivated listeners, situate melodic and harmonic movement, is the real object of musical study—the thing that needs to be understood in order to *understand music*. From this point of view even the three-dimensional pitch space explored by Longuet-Higgins is of little musical relevance, while Tymoczko's Möbius bands might just as well go and tie themselves in knots, for all that they tell us about music.

I have dwelt on Tymoczko's fourth claim, the one contained in the title to his book, for two reasons: first, because it gives rise to the illusion that musical order is a *secret* and that Tymoczko is now able to reveal what that secret is, and second, because its prominence distracts the reader from the real merits of his argument. The idea of a secret order of music is far from new, nor is it new to suggest that this order is geometrical. That was the master-thought of the Pythagorean cosmology and of the theory of the universe summarized by Ptolemy and accepted throughout the Western world until the scientific revolution. In a recent work that relies heavily on Lerdahl and Jackendoff (and on many other sources from psychology and cognitive science), Charles Nussbaum offers a similar "key to all the secrets," arguing that music supplies "plans of action": it provides "musical mental models" that "represent the features of the layouts and scenarios in which . . . virtual movements occur."[22] In another uncritical application of Lerdahl and Jackendoff, Diana Raffman uses the generative hypothesis to explain why the "secret meaning of music" is in fact an illusion, arguing that the syntax of music tempts us to attribute semantic significance to patterns that have no significance other than their musical form.[23] Tymoczko's is the latest in a series of books that promise more than they deliver, since they rely on theories whose application to music is largely wishful thinking. Longuet-Higgins, by contrast, seems to be getting somewhere, since his geometry clarifies distinctions between intervals that we hear but which are not easily represented in traditional notation. Moreover, his geometry is expressly directed toward providing a computational theory of tonal music—a theory that would show how musical objects and transitions might be represented in the nervous system.

The fourth claim of Tymoczko's argument, the claim to have revealed a hidden geometry of music, distracts us, I suggest, from his book's real merits. It is in his treatment of the other three claims that he makes his strongest case for the *musical constants* that anchor the extended common practice in the ordinary musical ear. He rightly argues that "for the foreseeable future, the majority of successful Western music will continue to exploit acoustic consonance, small melodic motions, consistent harmonies, clear tonal centers, and identifiable macroharmonies" (p. 392). He brushes aside serialism and makes a strong case for jazz and its offshoots as a refreshment of the old tonal principles, providing a new future for Western music beyond the decline of concert-hall listening.

Tymoczko's real purpose is to vindicate the grammar of the common practice not as a generative syntax, but as a form of "prolongation," to use the expression favored by Lerdahl and Jackendoff. His study of voice-led harmony in both the classical and the jazz traditions certainly succeeds in this. He makes some astute critical observations—especially in his discussions of Chopin and Debussy—but remains true to his purpose, which is to describe principles of musical organization that are parts of grammar rather than aesthetic effects. This leads to a certain non-judgmental vision of what is at stake in our musical tastes. For Tymoczko anything goes except deviant grammar. Not surprisingly, therefore, he mentions neither Theodor Adorno's

[22] Charles Nussbaum, *The Musical Representation* (Cambridge, MA: MIT Press, 2007), p. 82.

[23] Diana Raffman, *Language, Music and Mind* (Cambridge, MA: MIT Press, 1993).

critique of jazz as "musical fetishism" nor the apocalyptic vision of Thomas Mann's *Doktor Faustus*, and concludes his book with the very catholic hope that "there is music waiting to be written that combines the intellectuality of Bach (or Debussy) with the raw energy of Coltrane (or The Pixies or Einstürzende Neubauten)" (p. 395). Whatever you think of Einstürzende Neubauten, raw energy is not one of their leading characteristics, and I can only guess at the cultural pressures that led Tymoczko to conclude his book with a reference to that peculiarly depressing gang of nostalgic nihilists. Nevertheless, there is something right in Tymoczko's observation that all of the musics that he considers share the voice-led and prolongational structure of the common practice, and if we are to make distinctions among them (which surely we must) they must be made on grounds other than grammar. It is not grammar that distinguishes The Pixies from Elvis, but style, movement, and the quality of life.

Or is it so simple? When Adorno and Arnold Schoenberg argued that the tonal idiom had "become banal," they were not talking about *mere* syntax. They were referring to the way in which musical syntax is through and through subservient to its aesthetic employment. It is not Tymoczko's argument, therefore, that will offer the final reply to the modernists. For they were surely right to think that the common-practice grammar is something more than a grammar, and that it brings with it an emotional baggage acquired from centuries of use and maybe some decades of misuse, too. Consider "Here Comes Your Man," by the Pixies: perfect grammar, empty sentiment, both contained in a sequence of chords. It is not simply that we have heard this before. A simple melody harmonized over tonic and dominant can be pure, profound, and eternally fresh, like Schubert's "Wiegenlied." A piece laden with advanced harmonies, fabricated scales, and surprising cadences like Skryabin's seventh sonata can be shallow and stale by comparison. Skryabin was trying to escape one form of pollution—the obvious, the insincere, the banal—and fell victim to another. The Pixies have no such desire, but they certainly show why Skryabin ran away screaming from the kind of tonal thinking that prevails in pixieland.[24]

[24] I am grateful to Malcolm Budd, Wilfrid Hodges, Sarah-Jane Leslie, and Graeme Mitchison for comments and suggestions on an earlier version of this review.